Some 1d FreeFem++ examples

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Suppose we have a 1d pde on the [0,1] x-interval. and consider a uniform triangulation Th formed by one layer of triangles sitting on the [0,1] x-interval. Observe that:

1. Using functions that depend on two variables x and y when working on the border $[0,1] \times \{y = 0\}$ then $y = 0$ and the y variable is gone.

2. Using varf with the int1d(Th,1) function (assuming 1 is the label for the [0,1] x-interval) we can compute the correct entries of the mass matrix for the 1d problem and the entries to the right hand side.

3. All these values are stored in the matrix associated to the finite element space on the 2d triangulation but located at the degrees of freedom (dofs) that fall on the [0,1] x-interval and also in a vector associated to the right hand side for the 2d problem

4. The dofs of any finite element space defined on this triangulation Th that fall on the boundary [0,1] x-interval can be extracted from the total set of dofs.

5. Knowing these dofs the correct entries to construct the 1d mass matrix and 1d right hand side can be collected from the 2d mass matrix and right hand side.

6. One must be careful because the dofs for $\mathbb{P}_1$ are the nodes of the triangles but the dofs for $\mathbb{P}_2$ are the nodes of the triangles and the middle points of each edge.

7. For remeshing: we extend the 1d result to a 2d finite element function and use this one for remeshing the 2d mesh Th. After remeshing the 2d triangulation, the construction of the 1d mass matrix can be repeated.

1d steady convection-diffusion equation

First, we solve here the 1d steady convection-diffusion equation with uniform mesh and remeshing. The problem is

$$\begin{aligned} -\nu\, u_{xx} + b\, u_x &= 0, \quad x \in (0,1) \\ u(0) = 0,\ u(1) &= 1 \end{aligned}$$

using $\nu = 1/50$ and $b = 1$.

1d steady reaction-diffusion equation

We solve next the 1d reaction-diffusion equation with Dirichlet-Neumann boundary conditions. First with uniform mesh and then with remeshing. The problem is

$$\begin{aligned} -\nu\, u_{xx} + a_0\, u &= 0, \quad x \in (0,1) \\ u(0) = 1,\ u_x(1) &= 0 \end{aligned}$$

using $\nu = 1/10000$ and $a_0 = 1$. The code is similar and just needs change equation

Nonsteady 1d heat equation

We solve next the evolution 1d heat equation with uniform mesh. The problem is

$$\begin{aligned} \partial_t u &= u_{xx}, \quad x \in (0,1) \\ u(x,0 &= 1 - 2 * (x - 1/2) * sign(x - 1/2) \\ u(0,t) = 0,\ u(1,t) &= 0 \end{aligned}$$